MATHEMATICS PAPER IIA

TIME : 3hrs **Max. Marks.75**

Note: This question paper consists of three sections A,B and C.

SECTION A

VERY SHORT ANSWER TYPE QUESTIONS. 10X2 =20

1. Find two consecutive positive even integers, the sum of whose squares in 340.

2. If $1, -2$ and 3 are roots of $x^3 - 2x^2 + ax + 6 = 0,$ then find a.

3. Simplify –2i(3 + i)(2 + 4i)(1 + i) and obtain the modulus of that complex number.

4. Find the eccentricity of the ellipse whose equation is $|z-4| + \left|z - \frac{12}{5}\right| = 10$

5. all values of $1+i^{\,2/3}$

6. If ${}^{18}P_{r-1} : {}^{17}P_{r-1} = 9:7$, find r

7. Find the number of way of selecting a committee of 6members out of 10 members always including a specified member.

8. Find the term independent of x in the expansion of $\left(\frac{x^{1/2}}{3} - \frac{4}{x^2}\right)^{10}$

9. Find the constant C, so that $F(x) = C\left(\frac{2}{3}\right)^x, x = 1, 2, 3..........$ is the p.d.f of a discrete random variable X.

10. Find the mean deviation about the median for the following data.
    13, 17, 16, 11, 13, 10, 16, 11, 18, 12, 17

SECTION B

SHORT ANSWER TYPE QUESTIONS.

ANSWER ANY FIVE OF THE FOLLOWING **5 X 4 = 20**

11. Prove that $\frac{1}{3x+1}+\frac{1}{x+1}-\frac{1}{(3x+1)(x+1)}$ does not lie between 1 and 4 if x $\in$ R.

12. If n is a positive integer then show that

$$p+iq^{\frac{1}{n}} + p-iq^{\frac{1}{n}} = 2\ {p^2+q^2}^{\frac{1}{2n}} \cos\left\{\frac{1}{n} arc.\tan\frac{q}{p}\right\}$$

13. If the letters of the word PRISON are permuted in all possible ways and the words thus formed are arranged in dictionary order. Find the ranks of the word PRISON

14. Find the number of 4 digited numbers that can be formed using the digits 0, 1, 2, 3, 4, 5 which are divisible by 6 when repetition of the digits is allowed.

15. A fair die is rolled. Consider the events. A = {1, 3, 5}, B = {2, 3} and C = {2, 3, 4, 5}. Find

i) $P(A \cap B), P(A \cup B)$ ii) $P\left(\frac{A}{B}\right), P\left(\frac{B}{A}\right)$

iii) $P\left(\frac{A}{C}\right), P\left(\frac{C}{A}\right)$ iv) $P\left(\frac{B}{C}\right), P\left(\frac{C}{B}\right)$

16. A number x is drawn arbitrarily from the set {1, 2, 3, … 100}. What is the probability that $\left(x+\frac{100}{x}\right) > 29$.

17. Resolve $\frac{2x^2+3x+4}{(x-1)(x^2+2)}$ into partial fractions.

SECTION C

LONG ANSWER TYPE QUESTIONS.

ANSWER ANY FIVE OF THE FOLLOWING 5 X 7 =35

18. Solve $8x^4 - 2x^3 - 27x^2 + 6x + 9 = 0$ given that two roots have the same absolute value, but are opposite in signs

19. If n is an integer then show that

$$(1+\cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right).$$

20. For n = 0, 1, 2, 3, … n, prove that $C_0 \cdot C_r + C_1 \cdot C_{r+1} + C_2 \cdot C_{r+2} + \ldots + C_{n-r} \cdot C_n = {}^{2n}C_{n+r}$ and hence deduce that

i) $C_0^2 + C_1^2 + C_2^2 + \ldots + C_n^2 = {}^{2n}C_n$

ii) $C_0 \cdot C_1 + C_1 \cdot C_2 + C_2 \cdot C_3 + \ldots + C_{n-1}C_n = {}^{2n}C_{n+1}$

21. Find the sum of the infinite terms

$$\frac{5}{6\cdot 12} + \frac{5\cdot 8}{6\cdot 12\cdot 18} + \frac{5\cdot 8\cdot 11}{6\cdot 12\cdot 18\cdot 24} + \ldots\infty$$

22. Three boxes B_1, B_2 and B_3 contain balls detailed below.

	White	Black	Red
B_1	2	1	2
B_2	3	2	4
B_3	4	3	2

A die is thrown, B_1 is chosen if either 1 or 2 turns up, B_2 is chosen if 3 or 4 turns up and B_3 is chosen if 5 or 6 turns up. Having chosen a box in this way, a ball is chosen at random from this box. If the ball drawn is of red colour, what is the probability that it comes from box B_2?

23. Five coins are tossed 320 times. Find the frequencies of the distribution of number of heads and tabulate the result.

24. Find the mean and variance using the step deviation method of the following tabular data, giving the age distribution of 542 members.

Age in years (x_i)	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No.of members (f_i)	3	61	132	153	140	51	2

SOLUTIONS

1. Find two consecutive positive even integers, the sum of whose squares in 340.

Sol: 2n 2n + 2

$(2n)^2+(2n+2)^2=340$

$4n^2+4n^2+8n+4=340$

$8n^2+8n+4=340$

$2n^2+2n+1=85$

$2n^2+2n-84=0$

$n^2+n-42=0$

$(n+7)(n-6)=0$

$n=6$

12, 14 are two numbers.

2. If $1,-2$ and 3 are roots of $x^3-2x^2+ax+6=0,$ then find a.

Sol: $1,-2$ and 3 are roots of

$x^3-2x^2+ax+6=0$

$\Rightarrow 1\ \ -2\ \ +\ \ -2\ \ 3+3.1=a$

i.e., $a=-2-6+3=-5$

3. Simplify –2i(3 + i)(2 + 4i)(1 + i) and obtain the modulus of that complex number.

Sol: z = –2i(3 + i)(2 + 4i)(1 + i)

$=-2i(2+14i)(1+i)$

$=-2i(2+2i+14i-14)$

$=-2i(-12+16i)$

$=24i+32$

$=8(4+3i)$

$|z|^2=64\cdot 25$

$|z|=8\times 4=40$

4. Find the eccentricity of the ellipse whose equation is $|z-4|+\left|z-\frac{12}{5}\right|=10$

Sol.

SP+S^{1}P =2a

$S\ 4,0 \qquad S'\left(\frac{12}{5},0\right)$

2a = 10 $\Rightarrow$ a=5

SS'= 2ae

$$\Rightarrow 4-\frac{12}{5}=5X5e \Rightarrow \frac{8}{5}=10e \Rightarrow e=\frac{4}{5}$$

5. all values of $1+i^{\ 2/3}$

$$1+i^{\ 2/3}=\left[\left\{\sqrt{2}\left(\cos\frac{\pi}{4}+1\sin\frac{\pi}{4}\right)\right\}^2\right]^{\frac{1}{3}}$$

$$=\left\{2\left(\cos\frac{\pi}{2}+i\sin\pi/2\right)\right\}^{\frac{1}{3}}$$

$$=2^{\frac{1}{3}}\,cis\left(\frac{2k\pi+\frac{\pi}{2}}{3}\right) k=0,1,2$$

$$=2^{\frac{1}{3}}\,cis\ 4k+1\ \frac{\pi}{6}\ k=0,1,2$$

6. If ${}^{18}P_{r-1}:{}^{17}P_{r-1}=9:7$, find r

Sol: ${}^{18}P_{r-1}:{}^{17}P_{r-1}=9:7 \quad \Rightarrow \frac{{}^{18}P_{r-1}}{{}^{17}P_{r-1}}=\frac{9}{7}$

$$\Rightarrow \frac{18!}{\left[18-\ r-1\ \right]!}\times\frac{\left[17-\ r-1\ \right]!}{17!}=\frac{9}{7}$$

$$\Rightarrow \frac{18!}{19-r\ !}\ \frac{18-r\ !}{17!}=\frac{9}{7}$$

$$\Rightarrow \frac{18\times 17!\ 18-r\ !}{19-r\ \ 18-r\ !17!}=\frac{9}{7}$$

$$\Rightarrow 18\times 7=9\ 19-r$$

$$\Rightarrow 14=19-r \qquad \therefore r=19-14=5$$

7. Find the number of way of selecting a committee of 6members out of 10 members always including a specified member.

Sol:Since a specified member always includes in a committee, remaining 5 members can be selected from remaining 9 members in ${}^{9}C_{5}$ ways

$\therefore$ Required number of ways selecting a committee $= {}^{9}C_{5} = 126$

8. Find the term independent of x in the expansion of $\left(\frac{x^{1/2}}{3} - \frac{4}{x^2}\right)^{10}$

Sol. The general term in $\left(\frac{x^{1/2}}{3} - \frac{4}{x^2}\right)^{10}$ is

$$T_{r+1} = (-1)^r\, {}^{10}C_r \left(\frac{x^{1/2}}{3}\right)^{10-r} \left(\frac{4}{x^2}\right)^r$$

$$= (-1)^r\, {}^{10}C_r \left(\frac{1}{3}\right)^{10-r} (4)^r \cdot x^{5-\frac{r}{2}} \cdot x^{-2r}$$

$$= (-1)^r\, {}^{10}C_r \left(\frac{1}{3}\right)^{10-r} (4)^r \cdot x^{5-\frac{r}{2}-2r}$$

$$= (-1)^r\, {}^{10}C_r \left(\frac{1}{3}\right)^{10-r} (4)^r \cdot x^{\frac{10-5r}{2}} \quad \ldots 1$$

For the term independent of x,

Put $\frac{10-5r}{2} = 0 \Rightarrow 5r = 10 \Rightarrow r = 2$

Put r = 2 in eq.(1)

$$T_{2+1} = (-1)^2\, {}^{10}C_2 \left(\frac{1}{3}\right)^8 4^2 \cdot x^0$$

$$T_3 = \frac{80}{729}$$

9. Find the constant C, so that $F(x) = C\left(\frac{2}{3}\right)^x, x = 1, 2, 3..........$ is the p.d.f of a discrete random variable X.

Sol. Given $F(x) = C\left(\frac{2}{3}\right)^x, x = 1, 2, 3$

We know that $p(x)=C\left(\frac{2}{3}\right)^x, x=1,2,3\ldots$

$$\because \sum_{x=1}^{\infty} p(x)=1 \qquad \Rightarrow \sum_{x=1}^{\infty} c\left(\frac{2}{3}\right)^x=1$$

$$\Rightarrow c\left[\left(\frac{2}{3}\right)+\left(\frac{2}{3}\right)^2+\left(\frac{2}{3}\right)^3+........\infty\right]=1$$

$$\Rightarrow C\frac{2}{3}\left[1+\frac{2}{3}+\left(\frac{2}{3}\right)^2+........\infty\right]=1$$

$$\Rightarrow \frac{2c}{3}\left(\frac{1}{1-\frac{2}{3}}\right)=1 \Rightarrow \frac{2c}{3}\times 3=1 \Rightarrow c=\frac{1}{2}$$

10. Find the mean deviation about the median for the following data.
 13, 17, 16, 11, 13, 10, 16, 11, 18, 12, 17

Sol. Expressing the given data in the ascending order.
We get 10, 11, 11, 12, 13, 13, 16, 16, 17, 17, 18
Mean (M) of these 11 observations is 13.
The absolute values of deviations are $|x_i - M| = 3,2,2,1,0,0,3,3,4,4,5$

$$\therefore \text{ Mean deviation about Median } = \frac{\sum_{i=1}^{11}|x_i-M|}{n}=\frac{3+2+2+1+0+0+3+3+4+4+5}{11}$$

$$=\frac{27}{11}=2.45$$

11. Prove that $\frac{1}{3x+1}+\frac{1}{x+1}-\frac{1}{(3x+1)(x+1)}$ does not lie between 1 and 4 if $x \in R$.

Sol: $y=\frac{x+1+3x+1-1}{(3x+1)(x+1)}$

$$y=\frac{4x+1}{3x^2+4x+1}$$

$$3yx^2+x(4y-4)+y-1=0$$

Discriminant ≥ 0

$$4(y-1)^2-4\cdot 3y(y-1)\geq 0$$

$16(y-1)^2 - 12y(y-1) \geq 0$

$4(y-1)[4(y-1)-3y)] \geq 0$

$4(y-1)(y-4) \geq 0$

$(y-1)(y-4) \geq 0$

$\Rightarrow y \geq 4$ or $y \leq 1.$

12. If n is a positive integer then show that

$$(p+iq)^{\frac{1}{n}} + (p-iq)^{\frac{1}{n}} = 2\left(p^2+q^2\right)^{\frac{1}{2n}} \cos\left\{\frac{1}{n} arc.\tan\frac{q}{p}\right\}$$

Solution : -

Let $p+iq = r(\cos\theta + i\sin\theta)$

$r\cos\theta = p \quad r\sin\theta = q \quad \Rightarrow r^2 = p^2 + q^2$

$\therefore r = \sqrt{p^2+q^2}$

$\cos\theta = \dfrac{p}{\sqrt{p^2+q^2}} \sin\theta = \dfrac{q}{\sqrt{p^2+q^2}}$

$\tan\theta = \dfrac{q}{p} \Rightarrow \theta = \tan^{-1}\left(\dfrac{p}{q}\right)$

$(p+iq)^{\frac{1}{n}} + (p-q)^{\frac{1}{n}} = \left[r(\cos\theta + i\sin\theta)\right]^{\frac{1}{n}} + \left[r(\cos\theta - i\sin\theta)\right]^{\frac{1}{n}}$

$= r^{\frac{1}{n}}\left\{\cos\dfrac{\theta}{n} + i\sin\dfrac{\theta}{n} + \cos\dfrac{\theta}{n} - i\sin\dfrac{\theta}{n}\right\}$

$= \left(\sqrt{p^2+q^2}\right)^{\frac{1}{n}}\left\{2\cos\dfrac{1}{n}\theta\right\}$

$= 2\left(p^2+q^2\right)^{\frac{1}{2n}}\cos\left(\dfrac{1}{n}\tan^{-1}\dfrac{q}{p}\right)$

13. If the letters of the word PRISON are permuted in all possible ways and the words thus formed are arranged in dictionary order. Find the ranks of the word PRISON

Sol. The letters of the given word in dictionary order are

I N O P R S

In the dictionary order, first all the words that begin with I come. If I occupies the first place then the remaining 5 places can be filled with the remaining 5 letters in 5! ways.

Thus, there are 5! number of words that begin with I. On proceeding like this we get

I-------------> 5! ways

N------------->5! ways

O-------------> 5!ways

P I------------->4! ways

P N-------------> 4!ways

P O ------------->4! ways

P R I N------------->2 ! ways

P R I O------------->2! ways

P R I S N 0 -------------> 1 way

P R I S O N------------->1 way

Hence the rank of PRISON is

3 x 5! + 3 x 4! + 2 x 2! + 1 x1

= 360 + 72 + 4 + 1 + 1

= 438

14. Find the number of 4 digited numbers that can be formed using the digits 0, 1, 2, 3, 4, 5 which are divisible by 6 when repetition of the digits is allowed.

Sol. The first place of the number can be filled by any one of the given digits except '0' in 5 ways. The 2nd and 3rd places can be filled by any one of the given 6 digits in 6^2 ways.

x	x	x	x
5	6	6	1

After filling up the first 3 places, if we fill the units place with the given 6 digits, we get 6 consecutive positive integers. Out of these 6 consecutive integers exactly one will be divisible by '6'. Hence the units place can be filled in oneway.

$\therefore$ The number of 4 digited numbers formed using the given digits which are divisible by 6 when repetition is allowed. = $5 \times 6^2 \times 1 = 180$

15. A fair die is rolled. Consider the events. A = {1, 3, 5}, B = {2, 3} and C = {2, 3, 4, 5}. Find

i) $P(A \cap B), P(A \cup B)$ ii) $P\left(\frac{A}{B}\right), P\left(\frac{B}{A}\right)$

iii) $P\left(\frac{A}{C}\right), P\left(\frac{C}{A}\right)$ iv) $P\left(\frac{B}{C}\right), P\left(\frac{C}{B}\right)$

Sol. A fair die is rolled

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

$$P(B) = \frac{2}{6} = \frac{1}{3}$$

$$P(C) = \frac{4}{6} = \frac{2}{3}$$

$$n(S) = 6^1 = 6$$

Given A = {1, 3, 5}, B = {2, 3}, C = {2, 3,4,5}

i) $A \cap B = \{3\} P(A \cap B) = P\{3\} = \frac{1}{6}$

$$\therefore P(A \cap B) = \frac{1}{6}$$

$$P(A \cup B) = \{1, 2, 3, 5\}$$

$$n(A \cup B) = 4$$

$$n(S) = 6$$

$$P(A \cup B) = \frac{4}{6} = \frac{2}{3}$$

ii) $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{1/6}{1/3} = \frac{1}{2}$

$$P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)} = \frac{1/6}{1/2} = \frac{1}{3}$$

iii) $P\left(\frac{A}{C}\right) = \frac{P(A \cap C)}{P(C)} = \frac{2/6}{4/6} = \frac{1}{2}$

$\therefore$ A $\cap$ C = {3, 5}

$$P\left(\frac{C}{A}\right) = \frac{P(A \cap C)}{P(A)} = \frac{2/6}{3/6} = \frac{2}{3}$$

iv) $$P\left(\frac{B}{C}\right) = \frac{P(B \cap C)}{P(C)} = \frac{2/6}{4/6} = \frac{1}{2}$$

$\therefore$ B $\cap$ C = {2, 3}

$$P\left(\frac{C}{B}\right) = \frac{P(C \cap B)}{P(B)} = \frac{2/6}{2/6} = 1$$

16. A number x is drawn arbitrarily from the set {1, 2, 3, … 100}. What is the probability that $\left(x + \frac{100}{x}\right) > 29$.

Sol. Here the total number of cases is 100.

Let A be the event that x selected from the set {1, 2, 3, …100} has the property $x + \frac{100}{x} > 29$

Now $x + \frac{100}{x} > 29$

$\Leftrightarrow x^2 - 29x + 100 > 0$

$\Leftrightarrow (x-4)(x-25) > 0$

$\Leftrightarrow x > 25 \text{ or } x > 4$

$\Leftrightarrow x \in \{1, 2, 3, 26, 27, ...100\} = A(\text{say})$

so that the number of cases favourable to A is 78.

$\therefore$ The required probability $P(A) = \frac{78}{100}$

17. Resolve $\frac{2x^2 + 3x + 4}{(x-1)(x^2+2)}$ into partial fractions.

Sol. Let $\frac{2x^2 + 3x + 4}{(x-1)(x^2+2)} = \frac{A}{x-1} + \frac{Bx + C}{x^2 + 2}$

Multiplying with $(x - 1)(x^2 + 2)$

$2x^2 + 3x + 4 = A(x^2 + 2) + (Bx + C)(x - 1)$

$x = 1 \Rightarrow 2 + 3 + 4 = A(1 + 2)$

$9 = 3A \Rightarrow A = 3$

Equating the coefficients of x^2

$2 = A + B \Rightarrow B = 2 - A = 2 - 3 = -1$

Equating constants

$4 = 2A - C \Rightarrow C = 2A - 4 = 6 - 4 = 2$

$$\frac{2x^2+3x+4}{(x-1)(x^2+2)}=\frac{3}{x-1}+\frac{-x+2}{x^2+2}$$

18. Solve $8x^4-2x^3-27x^2+6x+9=0$ given that two roots have the same absolute value, but are opposite in signs

Sol: Suppose $\alpha, \beta, \gamma, \delta$ are the roots of the equation

$8x^4-2x^3-27x^2+6x+9=0$

i.e., $x^4-\frac{1}{4}x^3-\frac{27}{8}x^2+\frac{3}{4}x+\frac{9}{8}=0$ --------(1)

Sum of the root $\alpha+\beta+\gamma+\delta=\frac{1}{4}$ *and*

Product of the roots $\alpha\beta\gamma\delta=\frac{9}{8}$

Given $\beta=-\alpha \Rightarrow \alpha+\beta=0$

$\therefore 0+\gamma+\delta=\frac{1}{4} \Rightarrow \gamma+\delta=\frac{1}{4}$

Let $\alpha\beta=p, \gamma\delta=q$, so that $pq=\frac{9}{8}$

The equation having the roots

α, β is $x^2-\ \alpha+\beta\ \ x+\alpha\beta=0$

i.e., $x^2+p=0$

The equation having the roots

γ, δ is $x^2-\ \gamma+\delta\ \ x+\gamma\delta=0$

i.e., $x^2-\frac{1}{4}x+q=0$

from (1), (2) and (3)

$x^4-\frac{1}{4}x^3-\frac{27}{8}x^2+\frac{3}{4}+\frac{9}{8}=\ x^2+p$

$\left(x^2-\frac{1}{4}x+q\right)$

$\Rightarrow x^4-\frac{1}{4}x^3+\ \ p+q\ \ x^2-\frac{p}{4}x+pq$

Comparing the coefficients of x and constants

$\frac{-p}{4}=\frac{3}{4}\Rightarrow p=-3$

$pq=\frac{9}{8}\ \Rightarrow q=\frac{9}{8}\times\frac{-1}{3}=\frac{-3}{8}$

Substitute the value of p in eq(2)

$x^2-3=0$

$\Rightarrow x=\pm\sqrt{3}$

Substitute the value of q in eq(3)

$$x^2-\frac{1}{4}\times-\frac{3}{8}=0$$

$$\Rightarrow 8x^2-2x-3=0$$

$$\Rightarrow\ 2x+1\quad 4x-3\ =0$$

$$\Rightarrow x=-\frac{1}{2},\frac{3}{4}$$

$\therefore$ The roots of the given equation are

$$-\sqrt{3},-\frac{1}{2},\frac{3}{4},\sqrt{3}$$

19. If n is an integer then show that

$$(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right).$$

Sol. L.H.S. =

$$(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=$$

$$=\left(2\cos^2\frac{\theta}{2}+2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^n+$$

$$\left(2\cos^2\frac{\theta}{2}-2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)^n$$

$$=2^n\cos^n\frac{\theta}{2}$$

$$\left[\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)^n+\left(\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}\right)^n\right]$$

$$=2^n\cos^n\frac{\theta}{2}$$

$$\left(\cos\frac{n\theta}{2}+i\sin\frac{n\theta}{2}+\cos\frac{n\theta}{2}-i\sin\frac{n\theta}{2}\right)$$

$$=2^n\cos^n\frac{\theta}{2}\left(2\cos\frac{n\theta}{2}\right)$$

$$=2^{n+1}\cos^n\frac{\theta}{2}\cos\frac{n\theta}{2}=\text{R.H.S.}$$

20. For n = 0, 1, 2, 3, … n, prove that $C_0\cdot C_r+C_1\cdot C_{r+1}+C_2\cdot C_{r+2}+\ldots+C_{n-r}\cdot C_n={}^{2n}C_{n+r}$ and hence deduce that

i) $C_0^2+C_1^2+C_2^2+\ldots+C_n^2={}^{2n}C_n$

ii) $C_0\cdot C_1+C_1\cdot C_2+C_2\cdot C_3+\ldots+C_{n-1}C_n={}^{2n}C_{n+1}$

Sol. We know that

$(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n \ldots(1)$

On replacing x by 1/x in the above equation,

$$\left(1+\frac{1}{x}\right)^n = C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + ... + \frac{C_n}{x^n} ...(2)$$

From (1) and (2)

$$\left(1+\frac{1}{x}\right)^n (1+x)^n = \left(C_0 + \frac{C_1}{x} + \frac{C_2}{x^2} + ... + \frac{C_n}{x^n}\right)$$

$$(C_0 + C_1x + C_2x^2 + ... + C_nx^n) ...(3)$$

The coefficient of x^r in R.H.S. of (3)

$= C_0C_r + C_1C_{r+1} + C_2C_{r+2} + ... + C_{n-r}C_n$

The coefficient of x^r in L.H.S. of (3)

= the coefficient of x^r in $\frac{(1+x)^{2n}}{x^n}$

= the coefficient of x^{n+r} is $(1 + x)^{2n}$

$= {}^{2n}C_{n+r}$

From (3) and (4), we get

$C_0 \cdot C_1 + C_1 \cdot C_2 + C_2 \cdot C_3 + ... + C_{n-1}C_n = {}^{2n}C_{n+1}$

i) On putting r = 0 in (i), we get

$C_0^2 + C_1^2 + C_2^2 + ... + C_n^2 = {}^{2n}C_n$

ii) On substituting r = 1 in (i) we get

$C_0 \cdot C_1 + C_1 \cdot C_2 + C_2 \cdot C_3 + ... + C_{n-1}C_n = {}^{2n}C_{n+1}$

21. Find the sum of the infinite terms

$$\frac{5}{6\cdot12} + \frac{5\cdot8}{6\cdot12\cdot18} + \frac{5\cdot8\cdot11}{6\cdot12\cdot18\cdot24} + ...\infty$$

Sol. Let $S = \frac{5}{6\cdot12} + \frac{5\cdot8}{6\cdot12\cdot18} + \frac{5\cdot8\cdot11}{6\cdot12\cdot18\cdot24} + ...$

$$\Rightarrow 2S = \frac{2\cdot5}{1\cdot2}\left(\frac{1}{6}\right)^2 + \frac{2\cdot5\cdot8}{1\cdot2\cdot3}\left(\frac{1}{6}\right)^3 + \frac{2\cdot5\cdot8\cdot11}{1\cdot2\cdot3\cdot4}\left(\frac{1}{6}\right)^4 + ...$$

$$\Rightarrow 1 + \frac{2}{1}\left(\frac{1}{6}\right) + 2S = 1 + \frac{2}{1}\left(\frac{1}{6}\right) + \frac{2\cdot5}{1\cdot2}\left(\frac{1}{6}\right)^2 + \frac{2\cdot5\cdot8}{1\cdot2\cdot3}\left(\frac{1}{6}\right)^3 + ...$$

$$\Rightarrow \frac{4}{3}+2S = 1+\frac{2}{1}\left(\frac{1}{6}\right)+\frac{2\cdot5}{1\cdot2}\left(\frac{1}{6}\right)^2$$

$$+\frac{2\cdot5\cdot8}{1\cdot2\cdot3}\left(\frac{1}{6}\right)^3+......$$

Comparing $\frac{4}{3}+2S$ with $(1-x)^{-p/q}$

$$= 1+\frac{p}{1}\left(\frac{x}{q}\right)+\frac{p(p+q)}{1\cdot2}\left(\frac{x}{q}\right)^2+...$$

Here p = 2, q = 3, $\frac{x}{q}=\frac{1}{6}\Rightarrow x=\frac{q}{6}=\frac{3}{6}=\frac{1}{2}$

$$\therefore \frac{4}{3}+2S=(1-x)^{-p/q}=\left(1-\frac{1}{2}\right)^{-2/3}$$

$$=\left(\frac{1}{2}\right)^{-2/3}=(2)^{2/3}=\sqrt[3]{4}$$

$$\therefore 2S=\sqrt[3]{4}-\frac{4}{3}\Rightarrow S=\frac{\sqrt[3]{4}}{2}-\frac{2}{3}=\frac{1}{\sqrt[3]{2}}-\frac{2}{3}$$

$$\therefore \frac{5}{6\cdot12}+\frac{5\cdot8}{6\cdot12\cdot18}+\frac{5\cdot8\cdot11}{6\cdot12\cdot18\cdot24}+...=\frac{1}{\sqrt[3]{2}}-\frac{2}{3}$$

22. Three boxes B_1, B_2 and B_3 contain balls detailed below.

	White	Black	Red
B_1	2	1	2
B_2	3	2	4
B_3	4	3	2

A die is thrown, B_1 is chosen if either 1 or 2 turns up, B_2 is chosen if 3 or 4 turns up and B_3 is chosen if 5 or 6 turns up. Having chosen a box in this way, a ball is chosen at random from this box. If the ball drawn is of red colour, what is the probability that it comes from box B_2?

Sol. Let $P(E_i)$ be the probability of choosing the box B_i (i = 1, 2, 3).

Then $P(E_i) = \frac{2}{6}=\frac{1}{3}$; for i = 1, 2, 3

Having chosen the box B_i, the probability of drawing a red ball, say, $P(R/E_i)$ is given by

$$P\left(\frac{R}{E_1}\right)=\frac{2}{5},\ P\left(\frac{R}{E_2}\right)=\frac{4}{9} \text{ and } P\left(\frac{R}{E_3}\right)=\frac{2}{9}$$

We have to find the probability $P(E_2/R)$

By Bayer's theorem, we get

$$P\left(\frac{E_2}{R}\right) = \frac{P(E_2)P(R/E_2)}{P(E_1)P\left(\frac{R}{E_1}\right)+P(E_2)P\left(\frac{R}{E_2}\right)+P(E_3)P\left(\frac{R}{E_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{4}{9}}{\frac{1}{3}\left(\frac{2}{5}+\frac{4}{9}+\frac{2}{9}\right)} = \frac{\frac{4}{18}}{\frac{18+20+10}{5\times9\times3}} = \frac{5}{12}$$

23. Five coins are tossed 320 times. Find the frequencies of the distribution of number of heads and tabulate the result.

Sol. 5 coins are tossed 320 times

Prob. of getting a head on a coin

$$p = \frac{1}{2},\ n = 5$$

Prob. of having x heads

$$p\ \ x = x\ \ = {}^5C_x\ \ q\ \ ^5\left(\frac{1}{2}\right)^x$$

$$= {}^5C_x\left(\frac{1}{2}\right)^{5-x}\left(\frac{1}{2}\right)^x = {}^5C_x\left(\frac{1}{2}\right)^5\ \ x = 0,1,2,3,4,5$$

Frequencies of the distribution of number of heads = N.P(X = x)

$$= 320\left[{}^5C_x\left(\frac{1}{2}\right)^5\right];\ x = 0,1,2,3,4,5$$

Frequency of

Having 0 head = $320 \times {}^5C_0 \times \left(\frac{1}{2}\right)^5 = 10$

Having 1 head = $320 \times {}^5C_1 \times \left(\frac{1}{2}\right)^5 = 50$

Having 2 head = $320 \times {}^5C_2 \times \left(\frac{1}{2}\right)^5 = 100$

Having 3 head = $320 \times {}^5C_3 \times \left(\frac{1}{2}\right)^5 = 100$

Having 4 head = $320 \times {}^5C_4 \times \left(\frac{1}{2}\right)^5 = 50$

Having 5 head = $320 \times {}^{5}C_{5} \times \left(\frac{1}{2}\right)^{5} = 10$

N(H)	0	1	2	3	4	5
f	10	50	100	100	50	10

24. Find the mean and variance using the step deviation method of the following tabular data, giving the age distribution of 542 members.

Age in years (x_i)	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No.of members (f_i)	3	61	132	153	140	51	2

Sol.

Age in years C.I.	Mid point (x_i)	(f_i)	$d_i = \frac{x_i - A}{C}$ A = 55, C = 10	$f_i d_i$	d_i^2	$f_i d_i^2$
20-30	25	3	–3	–9	9	27
30-40	35	61	–2	–122	4	244
40-50	45	132	–1	–132	1	132
50-60	55→A	153	0	0	0	0
60-70	65	140	1	140	1	140
70-80	75	51	2	102	4	204
80-90	85	2	3	6	9	18
	N=542			–15	28	765

Mean $(\bar{x}) = A + \frac{\Sigma f_i d_i}{N} \times C = 55 + \frac{-15}{542} \times 10 = 55 - 0.277 = 54.723$

Variance (μ) = $\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2 = \frac{765}{542} - \left(\frac{-15}{542}\right)^2 = \frac{765}{542} - \frac{225}{(542)^2}$

$$= \frac{542 \times 765 - 225}{(542)^2} = \frac{414630 - 225}{293764} = \frac{414405}{293764} = 1.4106$$

$V(\mu) = V\left(\frac{X - A}{C}\right) = \left(\frac{1}{C}\right)^2 \cdot V(X) \quad \left[\because V(ax + n) = a^2 \cdot V(x)\right]$

$V(X) = C^2 \cdot V(\mu) = 100 \times 1.4106 = 141.06$.